Diffusion-Limited Aggregation

Dipankar Sarkar,Pulkit Gambhir

October 23, 2005

1 Introduction

Diffusion limited aggregation is a model for crystal growth or for the coalescing of dust particles. To start we would like to define some basic terms which are used throughout the assignment. Diffusion is the movement of particles due to temperature fluctuations and seen in Brownian motion. Diffusion can be simulated by a random walk on a computer. An aggregate is a collection of particles that are connected together. A growth process is called diffusion-limited when the aggregate increases in size by one particle at a time rather than by bunches. This happens since the density of particles is low and thus the particles do not come into contact with each other before reaching the aggregate.

2 The method

We start with a seed particle at the origin of a lattice. Another particle is allowed to walk randomly from far away until it is adjacent to the occupied site. It stops with a certain probability, called the sticking coefficient. If the particle does not stick, it moves on until it sticks. Then another particle is launched and moves until it is adjacent to the aggregate. Again it will stick only with a certain (same) probability (the stickiness factor) . This process repeats.

3 Fractals

To define what a fractal is, first consider the coastline of India. To measure the length of the coastline we need a ruler; with a meter stick, we measure the coastline and get a total length. Now with a smaller ruler, we measure it again and obtain a longer length. As we use smaller and smaller rulers, we obtain longer and longer lengths. If we use larger and larger rulers we obtain smaller and smaller total lengths. Because the coastline wiggles, the length is different when using different sized rulers. The coastline is considered to be a self-similar fractal if the length, as a function of the size of the ruler, follows a power law.

To determine the fractal dimension of the aggregate, first consider N(R), the number of particles that are closer than some distance R away from the center of mass. For a solid object in one dimension, the number is proportional to R. In two dimensions, this number is proportional to R^2. In three dimensions, this number is proportional to R^3.

Thus the number N(R) is proportional to R raised to a power which is the dimension. The function N(R) in Swiss cheese or other objects with holes, such as a fractal, obeys a power law relationship with a fractional power. This fractional power is the fractal dimension. To measure the fractal dimension, consider a log-log plot, that is a plot of the logarithm of N(R) versus the logarithm of R. The log-log plot should be linear and the slope of the line is the fractal dimension. For most objects, this relationship holds true over a finite range of R. At small R this relationship is limited by the size of the particles that make up the object. At large R this relationship is limited by the size of the object.

4 Algorithm

Below is the procedure that we used to create our computer program. Start by placing a seed particle in the middle of an area . The following steps are repeated until a big chunk/aggregation is formed; usually over 100000 steps. Place another particle some reasonable distance away from the aggregate. The particle diffuses (under goes a random walk) until it is next to the aggregate. . Then it will stick (the procedure stops) with certain probability; the sticking coefficient. A random number between zero and one is compared to the sticking coefficient. If the random number is larger than the sticking coefficient, the particle does not stick and it will move on until it does. In the program We have to make sure the particles do not pass through one another. If the particle wanders out of a certain area(the kill radius), the particle is discarded.

The fractal dimension is calculated next. For each particle, find the distance between it and the center of mass. All elements in the array with the index j greater than (or equal to) that distance are incremented by one. Plot the logarithm of the values of the array versus the logarithm of the index that corresponds to a distance. The resulting curve should be a line. The slope is the fractal dimension.

5 results

When we simulated the aggregate on a continium, the figure showed the formation with a sticking coefficient of 1.0. The fractal dimension for this aggregate is 1.64. This was comparable to the value of 1.66.

The fractal dimension of the aggregate grown with the sticking coefficient of 0.5 was 1.54 even though the aggregate looked denser than the one with sticking coefficient of 1.0. To see how the fractal dimension depends on statistical factors, we ran our program with a sticking coefficient of 0.5 twelve times. We calculated the average and found it to be 1.67, with a standard deviation of 0.08. The features seem to be thicker and coarser compared to the previous simulation even though the fractal dimension is virtually unchanged.

The features become even thicker as the sticking coefficient is decreased to 0.1. The trend continues until the features merge together and the aggregate becomes one big clump. The fractal dimension of the aggregate with a sticking coefficient of 0.1 is 1.57, is 1.92 with a sticking coefficient of 0.01, and is 1.94 with a sticking coefficient of 0.01. As you can already see that the fractal dimension approaches 2 as the sticking coefficient vanishes. At sticking coefficient 0.1, We calculated a low value for the fractal dimension. We believe that this low value is due to statistical fluctuations and we believe that if we had repeated it more number of times,would have gotten a different answer.

6 conclusion

Diffusion-limited aggregation is an irreversible process forming an aggregate of small particles. We ran computer simulations for a variety of sticking coefficients in continious two dimensions. We found that the fractal dimension does not change much until the sticking coefficient becomes less than 0.1. As the sticking coefficient vanishes the fractal dimension becomes close to the spatial dimension, i.e., close to 2. Since the process is statistical, we saw a large variation of fractal dimension when the sticking coefficient was 0.5.

Although diffusion limited aggregation is a model for crystal growth, it is not necessarily an accurate one; it only works in certain limits. The first problem is that diffusion limited aggregation does not include the effects of surface tension one way to include this feature would be to make a particle stick with a probability (different from one), called the sticking coefficient. We have included this effect in our calculations. Another problem with diffusion-limited aggregation as a model for crystal growth is that particles cannot detach. Also diffusion-limited aggregation only simulates the zero-density limit since the growth happens one particle at a time, rather than with bits containing a few or more particles. A way to fix this problem is to allow multiple aggregates that also diffuse and break up, but this has not been implemented in our program.

7 Graphs and plots

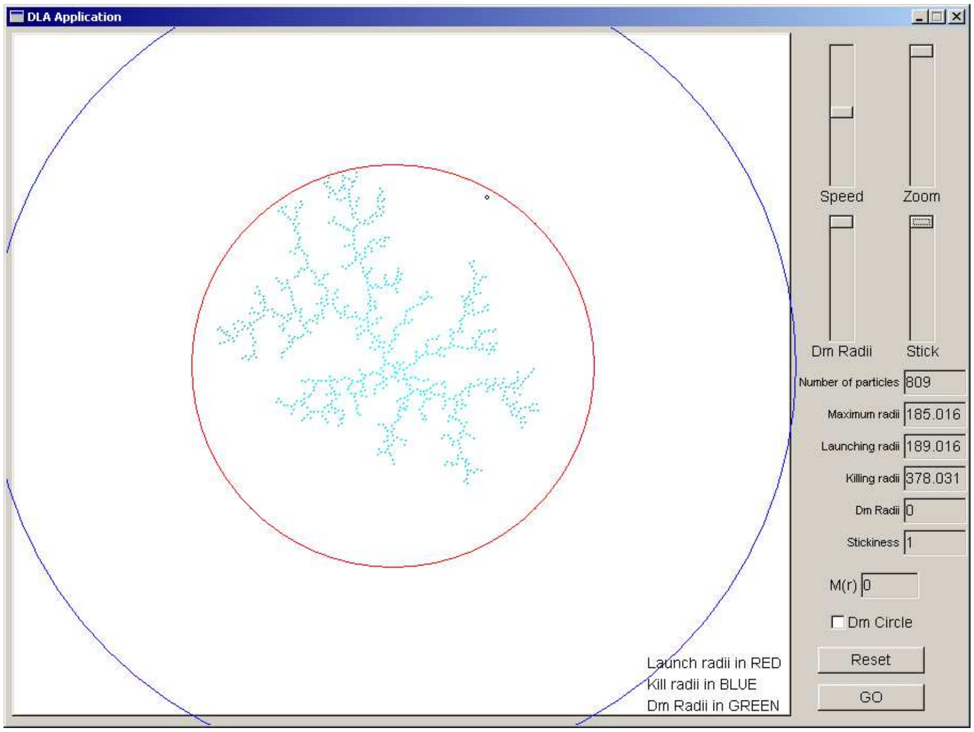

Figure 1: Stickiness factor=1.0

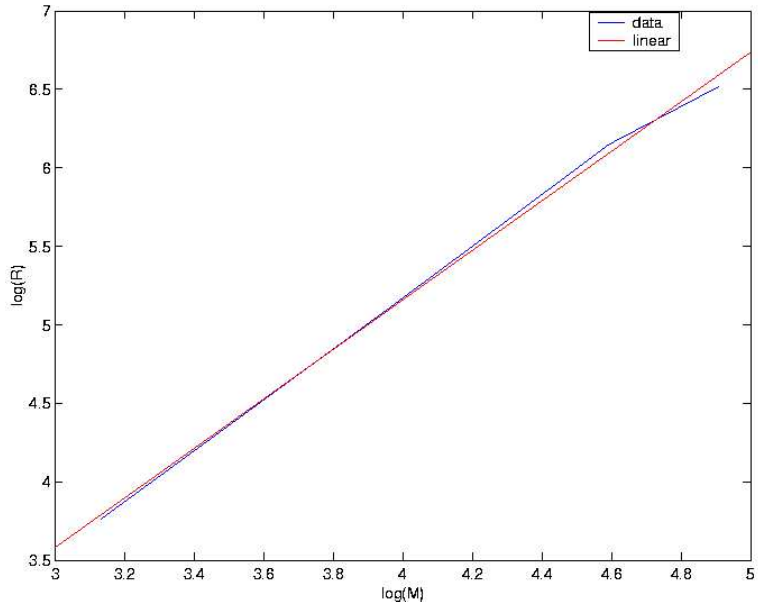

Figure 2: The $logM$ vs $logR$ plot, slope=1.64

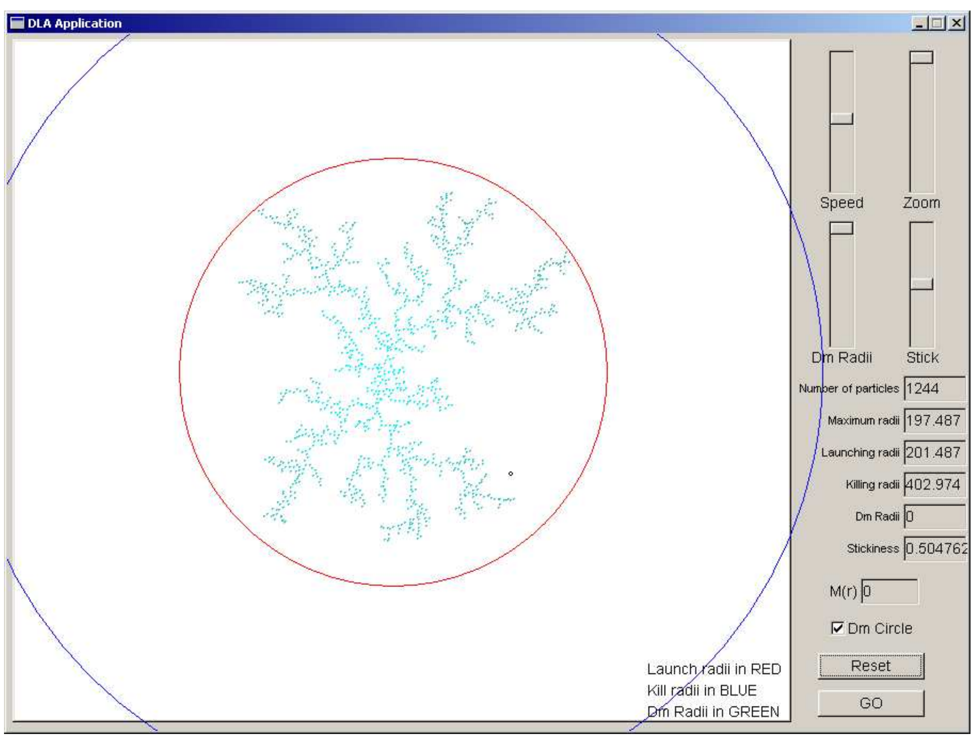

Figure 3: Stickiness factor=0.5

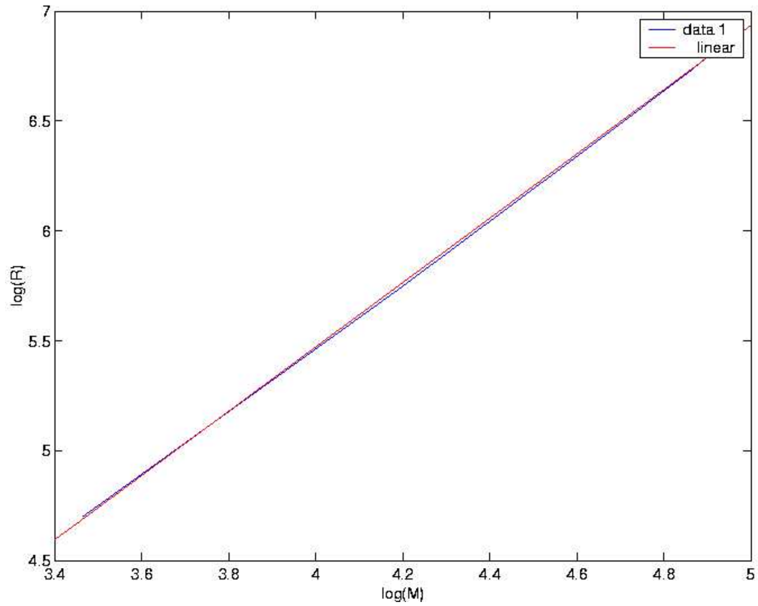

Figure 4: The $logM$ vs $logR$ plot,slope=1.63

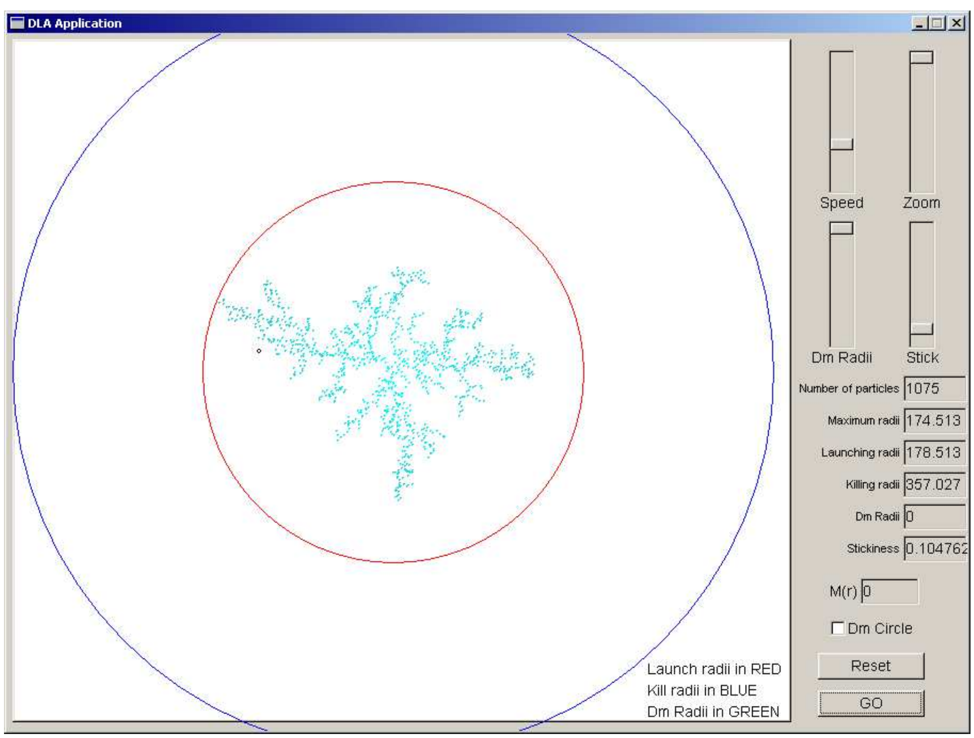

Figure 5: Stickiness factor=0.1

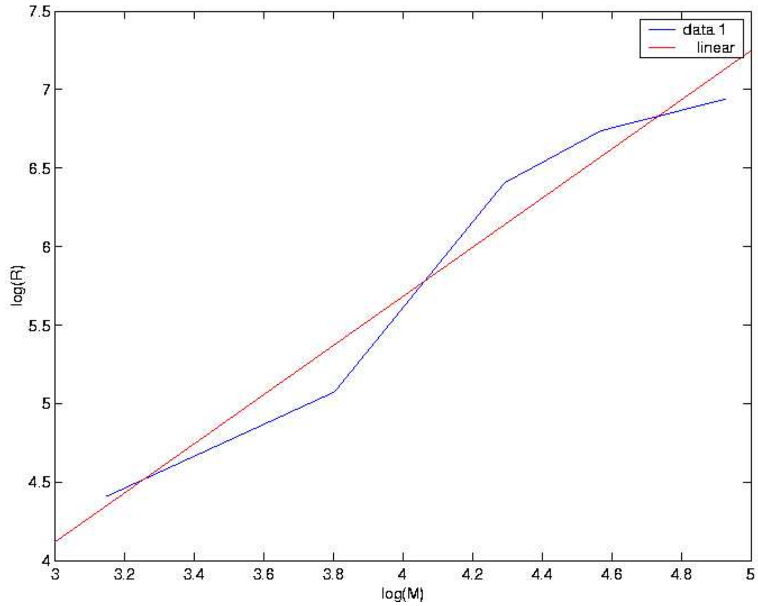

Figure 6: The $logM$ vs $logR$ plot,slope=1.92

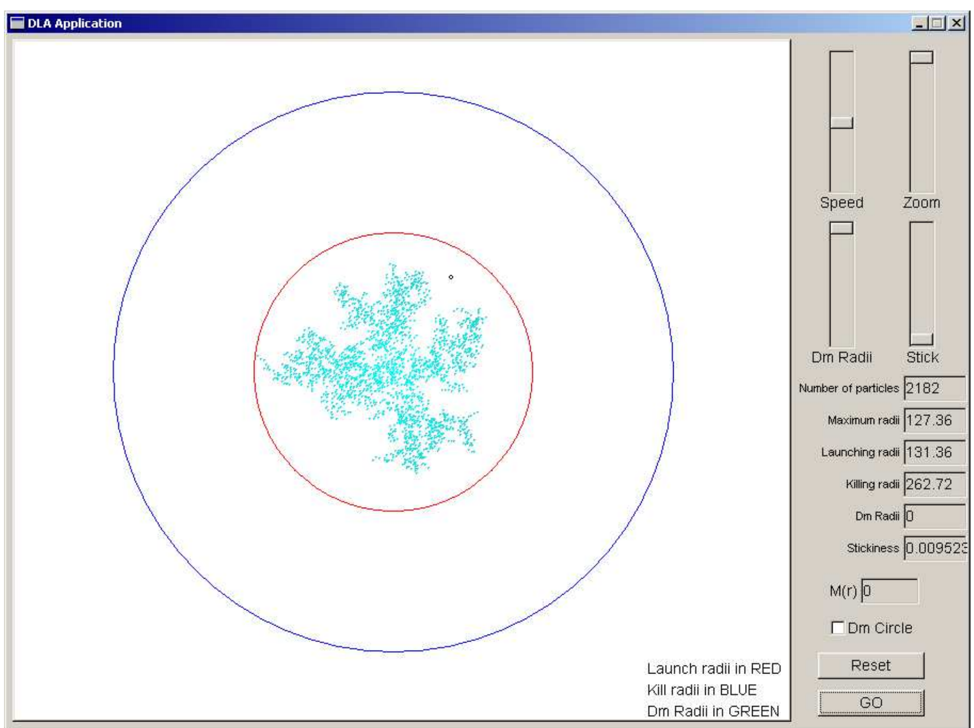

Figure 7: Stickiness factor=0.01

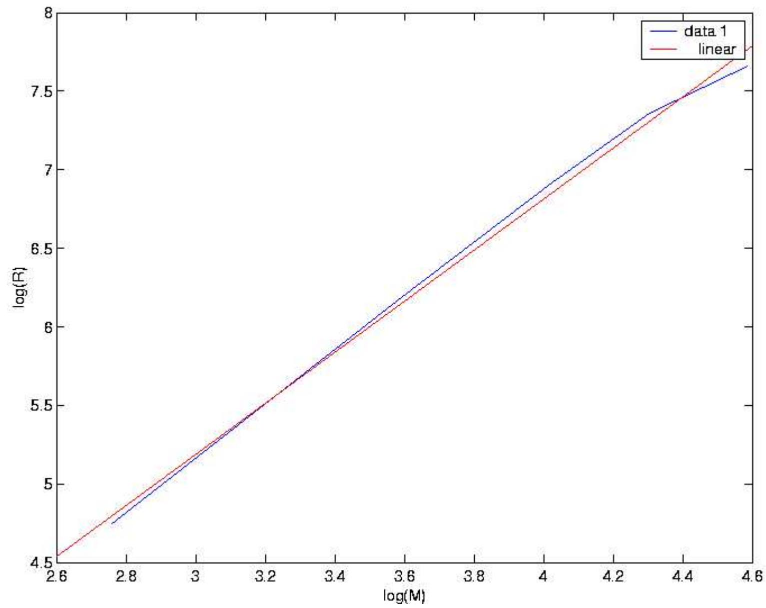

Figure 8: The $logM$ vs $logR$ plot,slope=1.94

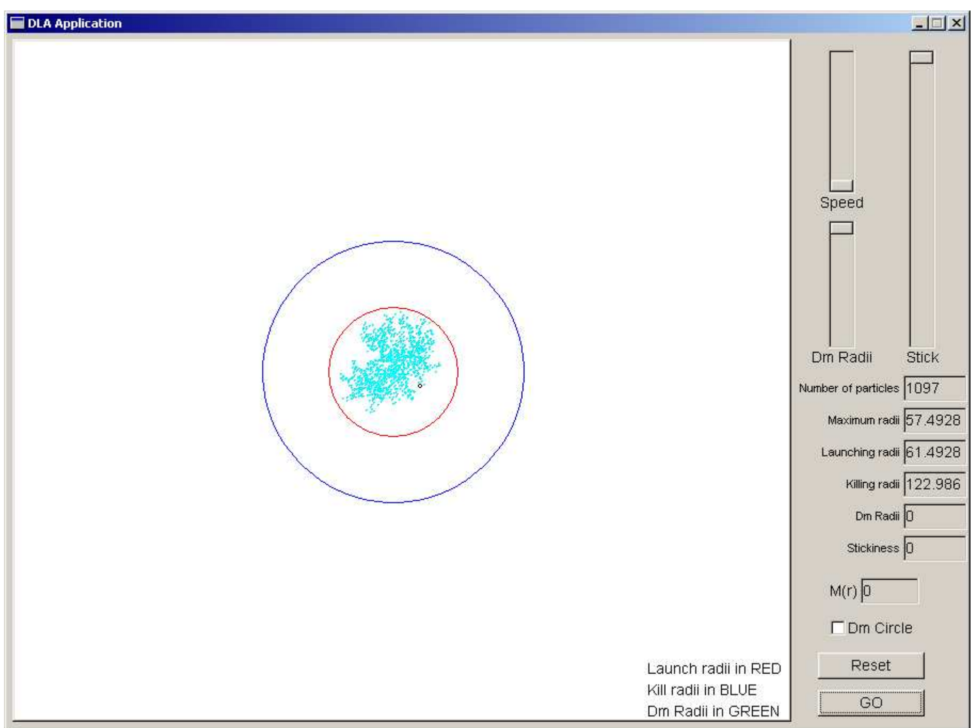

Figure 9: Stickiness factor=0.001

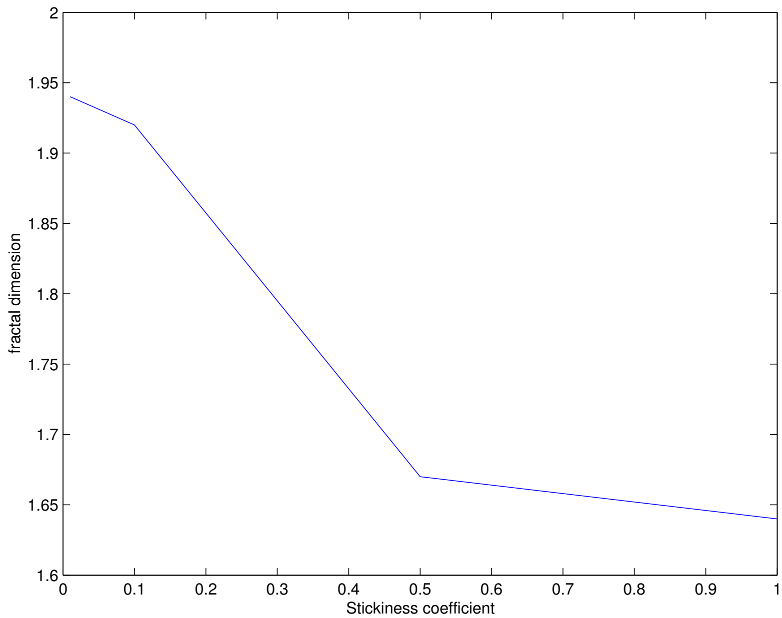

Figure 10: The stickiness factor vs fractal dimension plot